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Integral equivalence of permutation representations. (English summary)

Group theory (*Granville, OH*, 1992), 262–274, *World Sci. Publishing, River Edge, NJ*, 1993.

Let G be a finite group acting on two finite sets Ω and Ω'. If $\Omega \cong \Omega'$ as G-sets (i.e., a G-equivariant bijection exists between the two sets), then for any commutative ring R, the permutation modules $R\Omega$ and $R\Omega'$ are clearly isomorphic. Even for $R = \mathbf{Z}$ the converse is false, as shown by S. B. Conlon [J. Algebra 13 (1969), 496–508; MR 40 #5747], although his counterexamples are intransitive actions. In this striking article the author constructs for the first time a group G with transitive actions on Ω and Ω' for which $\mathbf{Z}\Omega \cong \mathbf{Z}\Omega'$ as $\mathbf{Z}G$-modules, but $\Omega \ncong \Omega'$ as G-sets. The construction was actually carried out over twenty years ago, one reason for the publication delay being the author's hope that the counterexample might be "parlayed into a counterexample for the group ring isomorphism problem, though that has not yet happened". The author takes $G = \mathrm{PSL}_2(29)$, with the isotropy subgroups in the two actions being non-conjugate subgroups isomorphic to the alternating group A_5. Two verifications of the counterexample are described, one involving hand computations and depending on the author's modular theory of permutation representations [in Representation theory of finite groups and related topics (Madison, WI, 1970), 137–144, Amer. Math. Soc., Providence, RI, 1971; MR 47 #8674], and the other using combinatorial considerations and machine computations to construct an explicit unimodular $|\Omega| \times |\Omega'|$-matrix. The author also takes the opportunity to discuss further counterexamples, present some general results and raise a number of interesting questions concerning $\mathbf{Z}$-equivalence of permutation modules, p-adic equivalence of permutation modules, and p-local subgroups.

{For the entire collection see MR 96d:20001}.

Reviewed by *Richard Lyons*