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Some new examples in 1-cohomology

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Abstract

This paper gives some new examples in the 1-cohomology theory of finite groups of Lie type, obtained from both computer calculations and the use of several theoretical results. In particular, the paper gives the first known examples of 1-cohomology groups of dimension greater than 2 for absolutely irreducible faithful modules of a finite group. The computer calculations were made originally while checking special cases of Lusztig's conjecture on characteristic p representations of algebraic groups, and we take this opportunity to announce in print some results in that direction. (They reinforce Lusztig's conjecture, even in a strong form suggested by Kato.)

Observed dimensions of 1-cohomology groups for finite groups have been remarkably small. This led Guralnick and Hoffman [GH, Conjecture 2] to make the following conjecture:

Conjecture 1. *Let V be a finite-dimensional faithful absolutely irreducible module for a finite group G. Then* $\dim H^1(G,V) \leqslant 2$.

This restated an earlier version by Guralnick [G, Conjecture 2], who had conjectured, under the same hypotheses, that "$\dim H^1(G,V) \leqslant \ell$, for some fixed ℓ (perhaps $\ell = 2$)." In this paper we present counterexamples to the above $\ell = 2$ conjecture, in the process exhibiting some methods which could illuminate the original, weaker conjecture that there was *some* absolute bound. While we would be pleasantly surprised if there was such a

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bound, it is not yet out of the question, and, in any case, we feel there is a lot in Guralnick's idea that 1-cohomology groups are, generally, remarkably small.

One central interest in 1-cohomology groups with irreducible coefficients V arises from their role in maximal subgroup theory [AS,S2], where the coefficients are a finite field F. For example, if F is a prime field, a maximal subgroup M is obtained from any complement to V in the semidirect product $V.G$ (of V with the group G acting on it). Such complements, up to conjugacy, are parameterized by elements of $H^1(G, V)$. The irreducible module V may be viewed as absolutely irreducible by replacing F with the finite field $\mathrm{End}_{FG}(V)$, and only a little more information is required to reduce the calculation to the case of a faithful action. Here, the best known general bounds are due to Guralnick and his collaborators [AG,G,GH], using the classification of finite simple groups. These bounds say that the dimension of $H^1(G, V)$ is at most a constant, currently $2/3$, times the dimension of V. In this form, the constant $2/3$ is sharp, but could presumably be lowered further by a more asymptotic formulation. However, the true asymptotic form of the growth rate is likely much smaller, and remains a mystery.

In lectures on this work, I have been asked what were the first examples where one even had a two-dimensional 1-cohomology group, since so many cases result in a dimension of one or zero. I do not know the very first case, but there is a family of dimension two examples, for finite orthogonal groups of type $D_{2\ell}$ in characteristic 2, in the 1975 paper [CPS1].

The present examples of larger dimension were found in the context of an entirely different investigation (reported briefly here in a later section), aimed at checking special cases of the Lusztig conjecture on representations in characteristic p. This work led us to incidentally calculate many dimensions of 1-cohomology groups, as coefficients, discussed below, of particular powers of q (always the highest power, when the coefficient in question is nonzero) in certain Kazhdan–Lusztig polynomials. More precisely, the polynomials involved are the parabolic Kazhdan–Lusztig polynomials of Deodhar [D], for the case of an affine Weyl group, with the ordinary Weyl group playing the parabolic subgroup role. These may also be interpreted as certain standard Kazhdan–Lusztig polynomials $P_{y,w}(q)$. In the Deodhar case y and w are the longest element in their respective right cosets of the ordinary Weyl group. For a given such polynomial, the coefficient of interest is that of $q^{(l(w)-l(y)-1)/2}$, which may be zero, but, when nonzero, is the coefficient of the largest power of q. Should these coefficients tend to infinity with increasing rank, a purely combinatorial issue, the methods of this paper would provide a counterexample to the weaker form of Guralnick's conjecture. In any case, if some information on the growth rate of these coefficients could be obtained, even for the affine type A case, it would give insight into the growth properties of the above 1-cohomology groups.

1. The theory behind the examples

Our first examples were established using the generic cohomology theory of [CPSK]. This is a "defining characteristic" theory for finite groups of Lie type, meaning that the representations involved have the same characteristic as that used to define the groups. The "large prime" version of the Lusztig conjecture, proved by [AJS], is also used in these first examples. Later, after a first version of this paper was written, it was realized that similar examples could be obtained in cross-characteristic, also called "nondefining characteristic," using the cross-characteristic generic cohomology theory of [CPS4], and the (much less difficult) "large prime" validity of the James conjecture, due to Geck–Gruber–Hiss, cf. [CPS4].

In either case, however, one must know validity of Lusztig's characteristic 0 conjecture for quantum groups of type A at a root of unity, known now by either work of Kazhdan–Lusztig [KL1,KL2,KL3,KL4], and Kashiwara and Tanisaki [KT1,KT2] for all types, or in type A by the somewhat more combinatorial "LLT theory," cf. Ariki's exposition [Ar2] of his earlier paper [Ar] and Leclerc's paper [LeT]. The works of Ariki and Leclerc also contain further references, and Leclerc especially cites Varagnolo and Vasserot [VV]. Curiously, both the Kashdan–Lusztig–Kashiwara–Tanisaki approach and the Ariki–Leclerc approach involve affine Lie algebras and perverse sheaves [BBD], though the perverse sheaves are on different spaces. The answers agree [Le], both expressible in terms of Kazhdan–Lusztig polynomials, through work of [LeT].

In each case—defining characteristic or cross-characteristic—the ultimate source of our examples is a coefficient in a Kazhdan–Lusztig polynomial for the affine Weyl group of type $\widehat{A_5}$. In the defining characteristic case, the underlying finite group is a $\mathrm{PSL}(6,q)$, with q a sufficiently large power of a sufficiently large prime p. The precise size requirement on p is unknown. If the original Lusztig conjecture is true, one may take $p=7$. The unknown size of p is a feature of the "large prime" Lusztig conjecture theory, not the generic cohomology theory, which gives specific bounds. Similarly, the size of the representation characteristic must be large, with size unknown, in the cross-characteristic case, though q could be a large power of a very small prime, even the prime 2. The cross-characteristic examples, however, appear to require representations of much larger rank groups, e.g., $\mathrm{PGL}(66,q)$. We will give details only in the defining characteristic case and just sketch the approach for cross-characteristic in a final section.

Let n be a fixed nonnegative integer and V a fixed finite-dimensional module for a reductive algebraic group $\overline{G}$ over $\overline{F}_p$. Then the n-cohomology groups $H^n(\overline{G}(q),V)$, over the group of F_q-rational points $\overline{G}(q)$ all have the same $\overline{F}_p$-dimension, for all sufficiently large powers q of p [CPSK]. (Parshall and Friedlander [FP, (3.2)] point out one may even take $q=p$, if p is—specifiably—sufficiently large.) Moreover, [CPS2], this dimension is at least the dimension of the space $\mathrm{H}^n(\overline{G},V)$. The latter space is isomorphic to $\mathrm{Ext}^n(\Delta(0),V)$ where $\Delta(0)$ is the one-dimensional trivial module, which happens also to be a Weyl module. Thus, if $V=L(\lambda)$ is irreducible with high weight λ, the dimension of this Ext space has an interpretation in terms of Kazhdan–Lusztig theory [CPS3].

In fact, if λ is a regular weight in the Jantzen region, and the Lusztig conjecture holds for $\overline{G}$, the dimension of this Ext space is a coefficient (see below) in a Kazhdan–Lusztig polynomial. (Cf. [S, p. 3], [A, (2.12)]; the result, a relatively easy property of parity conditions implied by the Lusztig conjecture, was first observed by Vogan in a category $\mathcal{O}$ context. A similar but more sophisticated calculation of the dimensions of spaces $\mathrm{Ext}^n(L(\mu),L(\lambda))$, is given in [CPS3, (3.6), (5.3), (3.9.1)]; note that the validity of the Lusztig conjecture is again assumed. The Lusztig conjecture is known by [AJS] to hold for all sufficiently large primes p, depending on the root system, though it is not known how large p must be.) If $n=1$, and $\mu\neq\lambda$ the coefficient involved is that of the highest power of the indeterminant $q=t^2$ (not to be confused with the prime power q).

We recall that a dominant weight λ is in the Jantzen region provided $(\lambda+\rho,\alpha^\vee)\leqslant p(p-h+2)$ for all positive roots α, where ρ is the sum of all fundamental weights, and h is the Coxeter number, which is n, for $\overline{G}=\mathrm{SL}(n.\overline{F_q})$. For our purposes it is sufficient to consider modules $L(\lambda)$ in the "principal block"—those with indexing weight λ of the form $w_0w(-\rho)-\rho$, with $\ell(w_0w)=\ell(w_0)+\ell(w)$. These weights are regular if $p\geqslant h$. The known results discussed above may be summarized precisely as follows:

Theorem 2. (1) *If λ is any dominant integral weight and n is any nonnegative integer, then we have*

$$\dim H^n\big(\overline{G},L(\lambda)\big)\leqslant \dim H^n\big(\overline{G}(q),L(\lambda)\big)$$

for all sufficiently large q, depending on λ and n.

(2) $\dim H^n(\overline{G},L(\lambda))$ *is the coefficient of $q^{(\ell(w_0w)-\ell(w_0)-n)/2}$ in the Kazhdan–Lusztig polynomial $P_{w_0,w_0w}(q)$, if $\lambda=w_0w.(-\rho)-\rho$ is in the Jantzen region, and p is large enough for the Lusztig conjecture to hold for $\overline{G}$.*

Deodhar [D] has shown that the Kazhdan–Lusztig polynomials required above can be computed using more tractable "parabolic" Kazhdan–Lusztig polynomials. Using Deodhar's approach and natural recursions, Chris McDowell, a student in the University of Virginia Research Experience for Undergraduates (REU) program (and supported also by NSF), wrote a computer program in the summer of 1998 to compute the polynomials P_{w_0y,w_0w} for $\mathrm{SL}(n,\overline{F_p})$, $n\leqslant 6$, with lengths additive in the expressions w_0y, w_0w as above. McDowell's results are available on my web page http://www.math.virginia.edu/~lls2l. We will quote from these results below. An independent program, of Anders Buch and Niels Lauritzen, which uses similar Kazhdan–Lusztig polynomial recursions, is available at http://home.imf.au.dk/abuch/dynkin/index.html, but it has apparently not yet been tested for the $\mathrm{SL}(6,\overline{F_p})$ case which leads to the counterexamples.

2. The examples in defining characteristic

The underlying finite groups will be of the form $\mathrm{SL}(6,q)$, with q a power of a prime $p\geqslant 7$. More accurately, we will use $\mathrm{PSL}(6,q)$, but first we will initially apply the theory in the SL case. Momentarily, we will assume p is large enough so that the Lusztig conjecture holds for this group, but it is convenient for notation to allow $p\geqslant 7$. (Of course all such primes are "large enough," if the original Lusztig conjecture is true.) The Kazhdan–Lusztig polynomials P_{w_0y,w_0w}, will be denoted also by the notation $P_{\mu,\lambda}$ when $\lambda=w_0w(-\rho)-\rho=w_0w.(-2\rho)$, and $\mu=w_0y.(-2\rho)$ are regular dominant (the weights λ,μ will change with p), and we write $\ell(\lambda)=\ell(w)$. Again, the notation $\ell(\lambda)$ depends on p, or more precisely, the p-alcove to which λ belongs. Also, this "length," as given here in terms of w, depends on our convention for fundamental reflection generators below, but it is also expressible in terms of alcove geometry (as the number of affine hyperplanes in the geometry separating λ from the weight 0).

The above conventions have been chosen partly to agree with the notation used by McDowell, whom we quote below, in his computer calculations. However, there is another issue we should discuss regarding the notation for Kazhdan–Lusztig polynomials, especially as they relate to the Lusztig conjecture. Lusztig, in stating his conjecture [L], wrote dominant weights in the form $-w(\rho)-\rho$, for some element w of the affine Weyl group. This weight may also be written $\widetilde{w}(-\rho)-\rho=\widetilde{w}.(-2\rho)$, where $w\mapsto\widetilde{w}$ is the automorphism fixing each ordinary Weyl group element, but taking a translation to its negative. The twist by the automorphism is often ignored in the literature—incorrectly, in some cases. (I am grateful to Jens Jantzen for alerting me to this issue, and explaining that such an inaccuracy occurs in his book [J, p. 294], copied in [CPS3, (5.2)] and in other papers.) If we want to ignore the twist, we can, by changing the generating set of fundamental reflections: View the generating reflections of the affine Weyl group, commonly viewed as reflections in hyperplanes containing the walls of a dominant alcove [J,CPS3], as instead occurring in hyperplanes containing the walls of an anti-dominant alcove. This changes a Kazhdan–Lusztig polynomial $P_{\tilde{y},\widetilde{w}}$ into $P_{y,w}$, and vice-versa. Consequently, when one uses these latter fundamental reflections and adjusts notation, Lusztig's conjectured character formula [L, (4)] reads, for $p\geqslant h$ and $w.–2\rho$ in the Jantzen region,

$$\operatorname{ch}L(w.-2\rho)=\sum_{y.-2\rho\,\text{dominant}}(-1)^{\ell(w)-\ell(y)}P_{y,w}(1)\operatorname{ch}\Delta(y.–2\rho).$$

We have already implicitly adopted this anti-dominant notation here, using it above in writing weight expressions such as $w_0w.(-2\rho)$, rather than $-\widetilde{w_0w}(\rho)-\rho$. We note that our polynomials $P_{\mu,\lambda}=P_{w_0y,w_0w}$ would still be denoted $P_{\mu,\lambda}$ in [CPS3], which makes little use of Coxeter group notation. (The same—correct—identification is, however, made in [CPS3, (5.3c)], following [A].)

Returning to $\overline{G}=\mathrm{SL}(6,\overline{F_q})$, we will let $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5$ denote the fundamental weights associated to the underlying root system.

Proposition 3 (McDowell). *Write Kazhdan–Lusztig polynomials as above, using representative weights in p-alcoves for $p=7$. (Thus $P_{y,w}$ is written $P_{\mu,\lambda}$ for $\mu=y.–2\rho$, $\lambda=w.–2\rho$.). Then there is an affine Weyl group element w with $w.–2\rho=\lambda=4\lambda_1+5\lambda_2+4\lambda_3+5\lambda_4+4\lambda_5$. If $\mu=0$ $(=w_0.–2\rho)$, we have*

$$P_{\mu,\lambda}(t^2)=1+8t^2+25t^4+51t^6+80t^8+87t^{10}+70t^{12}+38t^{14}+14t^{16}+3t^{18},$$

where t^2 is indeterminate (the usual "q"). The length $\ell(\lambda)=\ell(w)-\ell(w_0)$ defined above is 19.

Corollary 4. *Assume p is large enough for the Lusztig conjecture to hold for $\overline{G}=\mathrm{SL}(6,\overline{F_q})$. Put $V=L(w.–2\rho)$ with w as above. For all sufficiently large powers q of p, we have*

$$\dim H^1\big(\overline{G}(q),V\big)\geqslant 3,$$

Moreover, the center $Z(q)$ of $\overline{G}(q)$ acts trivially on V, and the dimension of $H^1(\overline{G}(q)/Z(q), V)$ is the same as that of $H^1(\overline{G}(q), V)$. The module V is a faithful absolutely irreducible module for the group $G = \overline{G}(q)/Z(q)$.

Thus, we have the main result of this paper:

Corollary 5. *The conjecture described at the beginning of this paper does not hold for the group $G = \overline{G}(q)/Z(q) = \mathrm{PSL}(6, q)$ for q any sufficiently large power of any sufficiently large prime p.*

We mention also the following result of McDowell, and its consequence through the CPS formula for Ext mentioned above.

Proposition 6 (McDowell). *Write Kazhdan–Lusztig polynomials as above, using representative weights in p-alcoves for $p = 7$. Then there are affine Weyl group elements w, y with $w.{-2\rho} = \lambda = 5\lambda_1 + 5\lambda_2 + 4\lambda_3 + 5\lambda_4 + 5\lambda_5$, and $y.{-2\rho} = \mu = \lambda_1 + 7\lambda_5$, and we have*

$$P_{\mu,\lambda}(t^2) = 1 + 9t^2 + 27t^4 + 49t^6 + 59t^8 + 44t^{10} + 21t^{12} + 4t^{14},$$

where t^2 is indeterminate (*the usual "q"*). *The lengths are $\ell(\lambda) = 20$, and $\ell(\mu) = 5$.*

Corollary 7. *With w and y as in the previous proposition, and $G = \mathrm{PSL}(6, q)$ for q any sufficiently large power of any sufficiently large prime p, we have*

$$\dim \mathrm{Ext}^1\big(L(w.{-2\rho}), L(y.{-2\rho})\big) \geqslant 4.$$

3. Some cases of the Lusztig conjecture

We take this opportunity to announce in print some computational results on the Lusztig conjecture. These results were obtained through work of another REU student, Mike Konikoff, with some input and modifications to his programs from me, especially to handle the case $p = 7$. (For computer calculations, the larger primes are often the most difficult to deal with.) Konikoff's programs enabled computing dimensions of weight spaces linked to the highest weight λ, for all restricted weights λ, for the group $\overline{G} = \mathrm{SL}(5, \overline{F_p})$, for $p = 5$. Konikoff's work was begun in 1993, and one of his programs, sufficient to determine the weight space dimensions, was completed in 1995. (Conference talks were given by Scott in Richmond in 1994 and Jerusalem in 1995 describing the work to this point.) Comparisons with actual Kazhdan–Lusztig polynomial predictions were made possible by McDowell's programs in 1997–1998. In the same period, Scott completed additional programs begun by Konikoff that allowed for independent cross checking of the results, and redesigned parts of the programs to allow all the same calculations for $p = 7$. (The case $p = 11$ is probably still inaccessible. The programs all work with so-called Baby Verma modules, which have dimension p^{10} in the $\mathrm{SL}(5, \overline{F_p})$ case.)

Independent calculations, completed in 1995, but not publicly announced until more recently, were done by Anders Buch and Niels Lauritzen at Aarhus for the case $p = 5$. All results of both teams agreed, and agreed with the conclusions of the Lusztig conjecture [L], when compared in 1998. Moreover, both calculations showed that the Lusztig's conjectured character formula held for all restricted weights (as high weight of the underlying irreducible module), though some of these weights were not in the Jantzen region. This had been suggested by Kato, in a paper [K] based on Lusztig's methods. Though the $p = 7$ case, which was inaccessible to Anders and Lauritzen, is more challenging computationally, the $p = 5$ case both teams did is more important, mathematically, because of Kato's formulation. The prime $p = 5$ is small enough so that $p \geqslant h$, which is also equal to 5 here, but $p = 5$ is not so large that all restricted weights lie in the Jantzen region, which requires $p \geqslant 2h - 3$. The group $\overline{G} = \mathrm{SL}(5, \overline{F}_p)$ is the smallest where one can have primes satisfying the first condition but not the second. Thus, with one nontrivial checked example in hand, Kato's formulation looks promising.

In addition, these small prime calculations tend to dispell the notion considered by some mathematicians, after the work of [AJS], that Lusztig's conjecture might really be only a very large prime phenomenon. Instead, it appears more likely that the character formula holds for all restricted weights, provided $p \geqslant h$, as suggested by Kato. These are the ones required to produce character formulas for all weights by Steinberg's famous tensor product formula [St].

Theorem 8. *For $p = 5$ or 7 and $\overline{G} = \mathrm{SL}(5, \overline{F}_p)$ Lusztig's conjectured character formula holds for all restricted weights. In particular, if $\lambda \in W_p.(-2\rho)$ is a restricted weight, then*

$$\operatorname{ch} L(\lambda) = \sum_{\mu \in W_p.(-2\rho)\ \text{dominant}} (-1)^{\ell(\lambda)-\ell(\mu)} P_{\mu,\lambda}(1) \operatorname{ch} \Delta(\mu).$$

Here $L(\lambda)$, $\Delta(\mu)$ denote the irreducible and Weyl modules with high weights λ, μ, respectively. Formulas for all other restricted weights are obtainable from Jantzen's translation principle [J, II,7].

4. Some cross-characteristic examples

After this paper was first written, the author realized that examples of large 1-cohomology groups could also be obtained in cross-characteristic. The cross-characteristic approach is analogous to that above in the defining characteristic case, but uses results from [CPS4], especially the cross-characteristic generic cohomology theory; this replaces the use of [CPSK] in the above defining characteristic arguments. The AJS "large prime" work is replaced by the much easier, but parallel, result of Geck–Gruber–Hiss, cf. [CPS4], for q-Schur algebras, q specialized to an ℓth root of 1. The rank of the underlying group is much bigger, however. It is still necessary to know decomposition numbers for q-Schur algebras at a root of unity (an ℓth root of unity here) in characteristic 0, or equivalently, for quantum enveloping algebras of type A. This can be supplied in one of two ways, as discussed in Section 2, but the only difference in the results used is that ℓ is a prime in the defining characteristic case, and may or may not be prime in the cross-characteristic case.

We will not go through the translation of our previous defining characteristic arguments to cross-characteristic in detail, but mention, without proof, an example which may be handled by the procedure. Perhaps, with the above ideas in mind, and a copy of [CPS4] in hand, the interested reader can fill in the arguments.

The underlying finite group is $G = \mathrm{GL}(66, q)$ or $\mathrm{PGL}(66, q)$. (It would also be possible to use $\mathrm{PSL}(66, q)$, using [CPS4, Theorem 10.5].) We take $\ell = 7$ and consider the modular irreducible unipotent representation $L = D(1, \lambda)$ associated to the partition $22, 18, 13, 9.4$. (This is the counterpart in partition language of the weight $w.\text{–}2\rho = 4\lambda_1 + 5\lambda_2 + 4\lambda_3 + 5\lambda_4 + 4\lambda_5$ in Proposition 3. Note that $22 = 4 + 5 + 4 + 5 + 4$, $18 = 5 + 4 + 5 + 4$, $13 = 4 + 5 + 4$, etc.) The prime p is sufficiently large dividing $q^\ell - 1$ $(= q^7 - 1$ here) but not dividing $q - 1$ (or the prime power q), so that [CPS4, Theorem 10.2] applies ($p > N(66)$ in the language of the latter reference). The only other requirement on q, beyond being a prime power, is that it allow a sufficiently large choice of p, satisfying the divisibility conditions of the previous sentence. Again, we conclude $H^1(G, L)$ has dimension $\geqslant 3$, as in Corollary 4. Also, L is faithful (if $G = \mathrm{PGL}(66, q)$) and absolutely irreducible, so also provides a counterexample to Conjecture 1.

The rank 66 arises as the sum $22 + 18 + 13 + 9 + 4$ of the partition parts. A somewhat smaller rank could be obtained by using $\ell = 6$ and recalculating $w.\text{–}2\rho$, interpreting w as a product of reflections from the affine Weyl group associated to $\ell = 6$, rather than $\ell = 7$. (We have not carried out the $\ell = 6$ calculation.)

Note that we required above that p not divide $q - 1$, as well as not divide q (the cross-characteristic requirement). This former requirement was necessary. We record here that the same approach, when p is a large prime dividing $q - 1$, does not yield cohomology groups of dimension greater than 1 for any partition λ of r, with $G = \mathrm{GL}(r, q)$. While the methods of [CPS4] do allow a combinatorial calculation of $\dim H^1(G, D(1, \lambda))$ in this case, even with just the size condition $p > r$, the answer is not in terms of Kazhdan–Lusztig polynomials. In fact, if λ is the partition dual to $r - 1, 1$, so that $L = D(1, \lambda)$ appears in (and is a summand of) the induction of the trivial module from the (maximal) parabolic subgroup associated to $r - 1, 1$, then one may continue the calculations of [CPS4, p. 71] to show $\dim H^1(G, D(1, \lambda)) = 1$. The same value 1 holds for λ dual to the trivial partition r of one part (so that $D(1, \lambda)$ is the trivial module), while for any other λ the dimension is 0.

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References

[A] H.H. Andersen, An inversion formula for the Kazhdan–Lusztig polynomials for affine Weyl groups, Adv. Math. 60 (2) (1986) 125–153.

[AJS] H.H. Andersen, J.C. Jantzen, W. Soergel, Representations of quantum groups at a pth root of unity and of semisimple groups in characteristic p: independence of p, Astérisque 220 (1994) 321.

[Ar] S. Ariki, On the decomposition numbers of the Hecke algebra of $G(m,1,n)$, J. Math. Kyoto Univ. 36 (4) (1996) 789–808.
[Ar2] S. Ariki, Representations of Quantum Algebras and Combinatorics of Young Tableaux. Translated from the 2000 Japanese edition and revised by the author, in: Univ. Lecture Ser., Vol. 26, Amer. Math. Society, Providence, RI, 2002, pp. viii+158.
[AG] M. Aschbacher, R. Guralnick, Some applications of the first cohomology group, J. Algebra 90 (2) (1984) 446–460.
[AS] M. Aschbacher, L. Scott, Maximal subgroups of finite groups, J. Algebra 92 (1) (1985) 44–80.
[BBD] A.A. Beilinson, J. Bernstein, P. Deligne, Faisceaux pervers, (French) Astérisque 100, 1982, pp. 5–171.
[CPS1] E. Cline, B. Parshall, L. Scott, Cohomology of finite groups of Lie type. I, Inst. Hautes Études Sci. Publ. Math. 45 (1975) 169–191.
[CPS2] E. Cline, B. Parshall, L. Scott, Detecting rational cohomology of algebraic groups, J. London Math. Soc. (2) 28 (2) (1983) 293–300.
[CPS3] E. Cline, B. Parshall, L. Scott, Abstract Kazhdan–Lusztig theories, Tohoku Math. J. (2) 45 (4) (1993) 511–534.
[CPS4] E. Cline, B. Parshall, L. Scott, Generic and q-rational representation theory, Publ. RIMS Kyoto 35 (1) (1999) 31–99.
[CPSK] E. Cline, B. Parshall, L. Scott, W. van der Kallen, Rational and generic cohomology, Invent. Math. 39 (2) (1977) 143–163.
[D] V.V. Deodhar, On some geometric aspects of Bruhat orderings, II, The parabolic analogue of Kazhdan–Lusztig polynomials, J. Algebra 111 (2) (1987) 483–506.
[FP] E.M. Friedlander, B.J. Parshall, Cohomology of infinitesimal and discrete groups, Math. Ann. 273 (3) (1986) 353–374.
[G] R.M. Guralnick, The dimension of the first cohomology group, in: Representation Theory, II, Ottawa, ON, 1984, in: Lecture Notes in Math., Vol. 1178, Springer-Verlag, Berlin, 1986, pp. 94–97.
[GH] R.M. Guralnick, C. Hoffman, The first cohomology group and generation of simple groups, in: Groups and Geometries, Siena, 1996, in: Trends Math., Birkhäuser, Basel, 1998, pp. 81–89.
[J] J.C. Jantzen, Representations of Algebraic groups, in: Pure Appl. Math., Vol. 131, Academic Press, Boston, MA, 1987, pp. xiv+443.
[KT1] M. Kashiwara, T. Tanisaki, Kazhdan–Lusztig conjecture for affine Lie algebras with negative level, Duke Math. J. 77 (1) (1995) 21–62.
[KT2] M. Kashiwara, T. Tanisaki, Kazhdan–Lusztig conjecture for affine Lie algebras with negative level. II. Nonintegral case, Duke Math. J. 84 (3) (1996) 771–813.
[K] S. Kato, On the Kazhdan–Lusztig polynomials for affine Weyl groups, Adv. Math. 55 (2) (1985) 103–130.
[KL1] D. Kazhdan, G. Lusztig, Tensor structures arising from affine Lie algebras, I, J. Amer. Math. Soc. 6 (4) (1993) 905–947.
[KL2] D. Kazhdan, G. Lusztig, Tensor structures arising from affine Lie algebras, II, J. Amer. Math. Soc. 6 (4) (1993) 949–1011.
[KL3] D. Kazhdan, G. Lusztig, Tensor structures arising from affine Lie algebras, III, J. Amer. Math. Soc. 7 (2) (1994) 335–381.
[KL4] D. Kazhdan, G. Lusztig, Tensor structures arising from affine Lie algebras, IV, J. Amer. Math. Soc. 7 (2) (1994) 383–453.
[Le] B. Leclerc, Decomposition numbers and canonical bases, Algebr. Represent. Theory 3 (3) (2000) 277–287.
[LeT] B. Leclerc, J.-Y. Thibon, Littlewood–Richardson coefficients and Kazhdan–Lusztig polynomials, in: Combinatorial Methods in Representation Theory, Kyoto, 1998, in: Adv. Stud. Pure Math., Vol. 28, Kinokuniya, Tokyo, 2000, pp. 155–220.
[L] G. Lusztig, Some problems in the representation theory of finite Chevalley groups, in: The Santa Cruz Conference on Finite Groups, Univ. California, Santa Cruz, CA, 1979, in: Proc. Sympos. Pure Math., Vol. 37, Amer. Math. Society, Providence, RI, 1980, pp. 313–317.
[S] L. Scott, Representations in characteristic p, in: The Santa Cruz Conference on Finite Groups, Univ. California, Santa Cruz, CA, 1979, in: Proc. Sympos. Pure Math., Vol. 37, Amer. Math. Society, Providence, RI, 1980, pp. 313–317.
[S2] L. Scott, Linear and nonlinear group actions, and the Newton Institute program, in: Algebraic Groups and their Representations, Cambridge, 1997, in: NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., Vol. 517, Kluwer, Dordrecht, 1998, pp. 1–23.

[St] R. Steinberg, Representations of algebraic groups, Nagoya Math. J. 22 (1963) 33–56.
[VV] M. Varagnolo, E. Vasserot, Standard modules of quantum affine algebras, Duke Math. J. 111 (3) (2002) 509–533.